Superposition Principle

It states that the interaction between any two charges is completely unaffected by the presence of other charges.

* If several point charges $q_1, q_2, q_3 \ldots$ ~~simultaneously exert electric force on the charge q, then the~~ located respectively, at points with position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3 \ldots$, the resultant force $\vec{F}$ on a charge q located at point $\vec{r}$ is the vector sum of the forces exerted on q by each of the charges $q_1, q_2, q_3 \ldots$

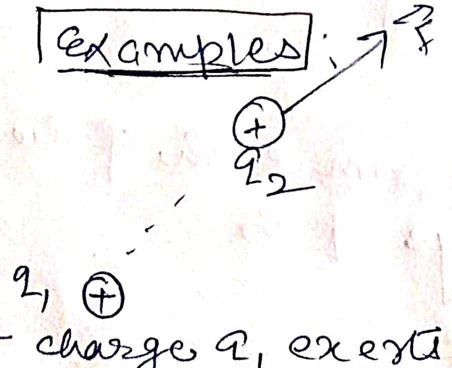
* So, mathematically it can be represented as:

$$\vec{F} = K \frac{q_1 q (\vec{r} - \vec{r}_1)}{|\vec{r} - \vec{r}_1|^3} + K \frac{q q_2 (\vec{r} - \vec{r}_2)}{|\vec{r} - \vec{r}_2|^3} + \ldots$$

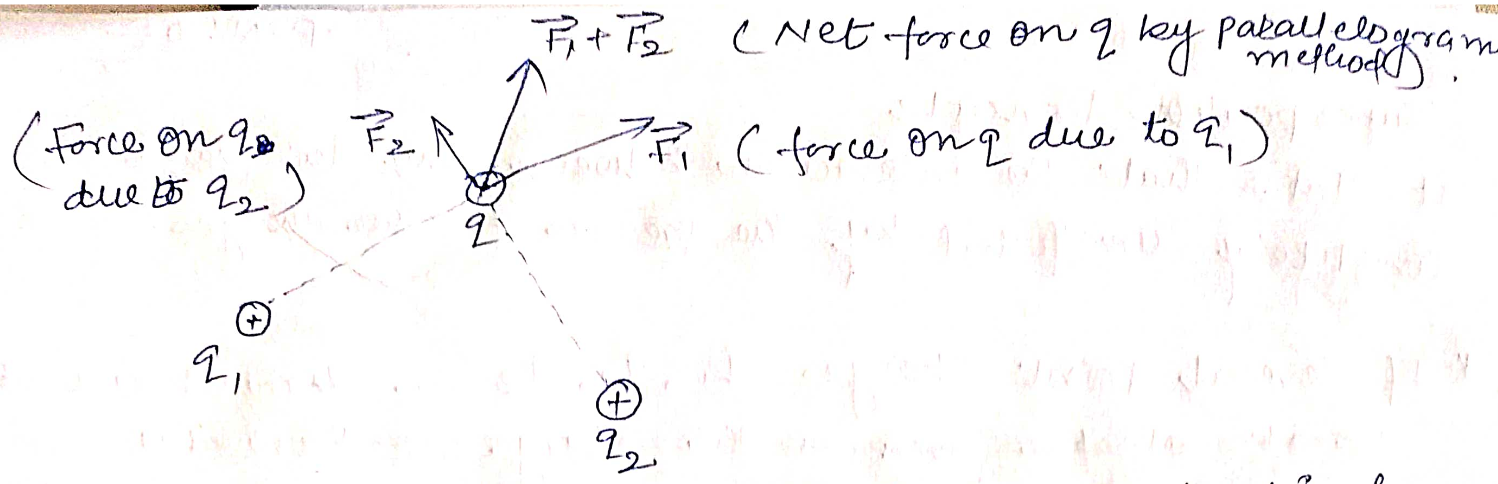
$$\vec{F} = Kq \sum_{i=1}^{N} \frac{q_i (\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3}$$

where $K = \frac{1}{4\pi\epsilon_0}$

Examples:



A charge q_1 exerts an electric force $\vec{F}$ on charge q_2.



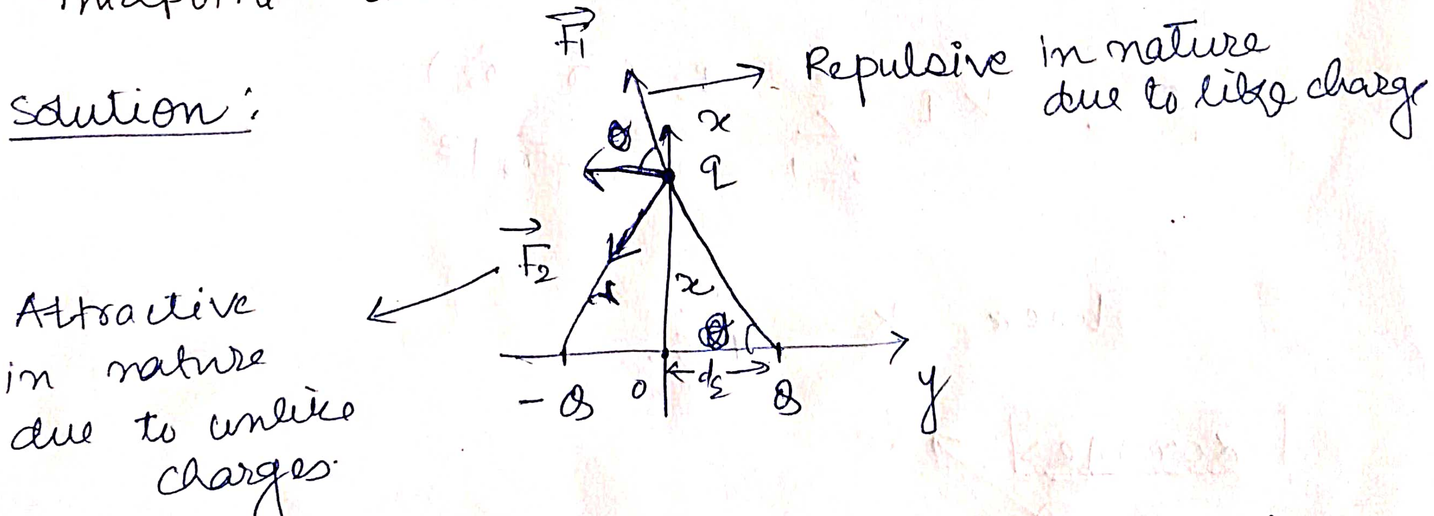
Two point charges q_1 and q_2 exerts electric force $\vec{F_1}$ and $\vec{F_2}$ on point charge q. The net force on q is vector sum of these forces.

Example

Problem:

Two point charges +Q and -Q are separated by a distance d. A positive charge q is equidistant from these charges, at a distance x from their midpoint. What is the electric force $\vec{F}$ on q?

Solution:



From geometry the distance from each of the charges +Q and -Q to charge q is

$$r = \sqrt{x^2 + d^2/4}$$

Magnitude of individual coulomb forces exerted by $+Q$ and $-Q$ are

$$\cancel{F_1 \propto F_2 \propto = \frac{1}{4\pi\epsilon_0} qQ}$$

$$F_1 = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{(x^2 + d^2/4)}$$

$$F_2 = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{(x^2 + d^2/4)}$$

vertical component of $\vec{F_1}$ and $\vec{F_2}$ will cancel each other. $F_x = F_1 \sin\theta - F_2 \sin\theta$

Horizontal component of $\vec{F_1}$ and $\vec{F_2}$ will be added

$$F_y = F_1 \cos\theta + F_2 \cos\theta$$

$$= 2 \frac{1}{4\pi\epsilon_0} \frac{qQ}{(x^2 + d^2/4)} \cos\theta$$

$$\cos\theta = \frac{d/2}{(x^2 + d^2/4)^{1/2}}$$

$$F_y = \frac{2}{4\pi\epsilon_0} \frac{qQ\, d/2}{(x^2 + d^2/4)^{3/2}}$$

when $x >> d$

then $F_y = \frac{1}{4\pi\epsilon_0} \frac{qQd}{x^3}$